

MATHEMATICS

Time allowed: 1 hour 30 minutes

- All answers (including any diagrams, graphs or sketches) should be written on paper, and scanned into a **single** PDF file. Graph paper is not required.
 - Answer **all** questions in Section A and **two** questions from Section B.
 - Candidates are permitted to use calculators, provided they comply with A level examining board regulations. They must be made available on request for inspection by invigilators, who are authorised to remove any suspect calculators.
 - Statistical tables will be provided. Note that the tables refer to the **right-hand** tails of the distributions, that is, probabilities of the form $p = \mathbb{P}(X \geq x)$ where X is a random variable and x an **upper** percentage point of its distribution.
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Section A

1. Simplify the following expressions as far as possible, showing your working clearly.

(a) $\frac{1}{1 - \frac{3}{3+x}} - \frac{3}{x}$ [3 marks]

(b) $\frac{\frac{4}{\sqrt{5}} + 5\sqrt{2}}{2\sqrt{5} - \sqrt{2}} - 3\sqrt{\frac{2}{5}}$ [5 marks]

(c) $\log_2 3 + \log_2 14 - \log_2 21$ [4 marks]

2. Absorption of alpha-particles by solid material is modelled as follows: the number of particles passing through a layer of thickness d cm is $N = Ae^{-\mu d}$ where A and μ are constants. (The particles not passing through are absorbed.)

(a) Explain the meaning of the constant A in this context. [1 mark]

(b) A 2 cm layer of a material was found to absorb 40% of the particles. Find the value of μ for this material, to four decimal places. [4 marks]

(c) How thick should the layer in (b) be to absorb 95% of the particles? [3 marks]

3. Solve the equation

$$12(\cos \theta + 1) \cos \theta + 5 \sin^2 \theta = 0$$

for values of θ between 0° and 360° . [6 marks]

4. In the binomial expansion of $(a - 3x)^9$ in powers of x , the coefficient of x^5 is equal to -378 . Find the value of a . [4 marks]

5. A cubic polynomial is given by $f(x) = (2x - a)(8x^2 - 2x - 3)$, where a is a constant. The curve $y = f'(x)$ intersects the y -axis at the point $(0, -1)$.

(a) Find the value of a . [4 marks]

(b) The curve $y = f(x)$ intersects the x -axis at three points. Find the x -coordinates of these points. [4 marks]

(c) The curve $y = f'(x)$ intersects the line $y = -1$ at two points. Find the x -coordinates of these points. [3 marks]

(d) Find the area enclosed by the curve $y = f'(x)$ and the line $y = -1$. [4 marks]

6. (a) Given that $y = \frac{4}{x} + \frac{1}{5}x^{\frac{5}{2}} + x$, find the value of $\frac{dy}{dx}$ at $x = 2$. [3 marks]

(b) Find the indefinite integral $\int (4x^{\frac{1}{3}} - 3x^{-\frac{2}{5}})dx$. [2 marks]

7. Vectors \mathbf{u} and \mathbf{v} are defined by $\mathbf{u} = 2\mathbf{i} - \mathbf{j}$ and $\mathbf{v} = \mathbf{i} + 2\mathbf{j}$.

(a) Find, in terms of \mathbf{i} and \mathbf{j} , the vector $\mathbf{v} - 2\mathbf{u}$, and calculate $|\mathbf{v} - 2\mathbf{u}|$. [3 marks]

(b) Determine the range of values for μ such that $\mu|\mathbf{v} - 2\mathbf{u}| < |\mathbf{v}| - 2|\mathbf{u}|$. [3 marks]

(c) Write the vector $a\mathbf{u} + b\mathbf{v}$ in terms of \mathbf{i} and \mathbf{j} , and determine the values of a and b such that $a\mathbf{u} + b\mathbf{v} = \mathbf{i} + \mathbf{j}$. [4 marks]

Section B

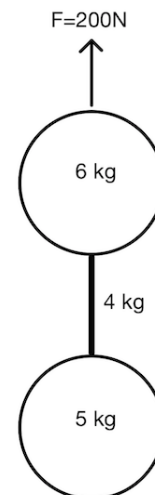
8. Circle C on the xy -plane has centre A and satisfies the equation

$$x^2 + y^2 + k^2 = 3 + 2(x + ky),$$

where k is a constant. The origin O lies on C , and the line L tangent to C at O has positive slope.

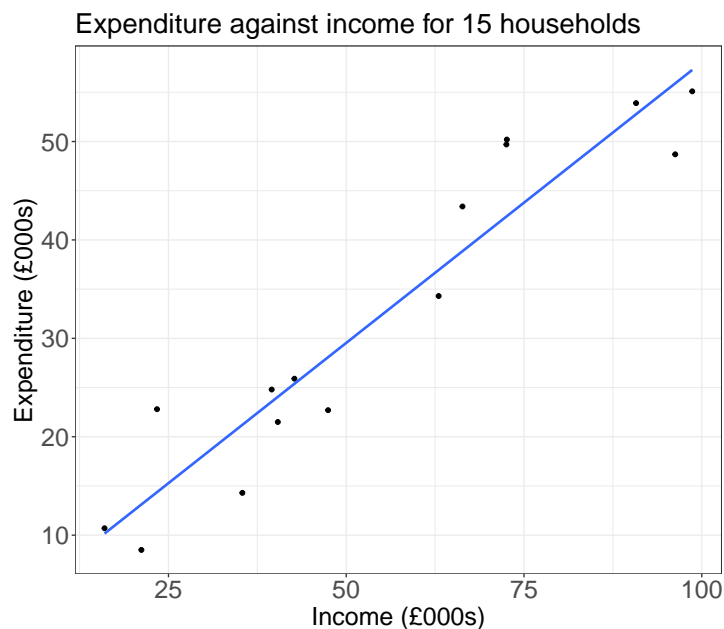
- (a) Determine (i) the radius of C (ii) the value of k , (iii) the coordinates of A , and (iv) the equation of L . [9 marks]
 - (b) The circle C intersects the x -axis also at a point P different from O . Find the x -coordinate of P . [2 marks]
 - (c) The line tangent to C at P intersects L at a point D . Find the coordinates of D . [4 marks]
 - (d) What percentage of the area of the triangle ODP lies inside the circle C ? [5 marks]
9. Two masses weighing 6kg and 5kg are connected to each other by a vertical uniform rod weighing 4kg, as shown in the diagram. An upward force of 200N is applied on the 6kg mass.

- (a) For each of the three objects (6kg mass, 4kg rod, 5kg mass), sketch a diagram showing all the forces acting on the object. [8 marks]
- (b) Find the acceleration of the system. Acceleration due to gravity should be taken as 9.81ms^{-2} . [3 marks]
- (c) Find the tension force at the top of the rod. [4 marks]
- (d) Find the tension force at the midpoint of the rod. [5 marks]



10. (a) In a factory producing lengths of wire, faults occur in the wire at a mean rate of 0.4 per 1000m. A Poisson distribution is used to model the number of faults in a particular length of wire.
- (i) Find the probability of two or more faults in 1000m of wire. [2 marks]
 - (ii) Find the probability of three or fewer faults in 8000m of wire. [3 marks]
 - (iii) For what length of wire (to the nearest metre) is the probability of zero faults equal to 0.3? [3 marks]
 - (iv) A new wire manufacturing procedure is tested in the factory. After the introduction of the new procedure, 2 faults are observed in a particular 10000m length of wire.
Stating your hypotheses clearly, and explaining any notation you use, test at the 5% level whether there is any evidence that the new manufacturing procedure has reduced the rate of faults. [5 marks]

- (b) In an economic survey of 15 households, the following graph was produced of expenditure (in thousands of pounds) against income (in thousands of pounds):



The regression equation is

$$y = 1.04 + 0.57x,$$

where y is expenditure and x is income.

- (i) Describe the relationship between income and expenditure. [2 marks]
- (ii) Interpret the gradient and intercept terms in this context. [2 marks]
- (iii) Calculate the predicted expenditure for a household with an income of £150000. Explain why this estimate should be treated with caution. [3 marks]